Roundabout Capacity Analysis Based on Conflict Technique

By Ning Wu¹ and Werner Brilon

Abstract

The conventional way of capacity analysis is based on the critical gap method or on empirical regression or a combination of both. Influence of pedestrians is modeled by reduction factors with a rather questionable empirical background. Thus, the current methods are based on an incoherent mix of sophistications. Moreover, they do not account for the interaction between the different elements.

The paper presents a new model which treats the whole intersection as one entity. Here all the conflicts where different streams (vehicles and non-motorized road users) intersect within the roundabout are identified. Each conflict point is treated as one queuing system with a simplified queuing mechanism. This theory has already been used successfully for 4-way-stop (4WSC) and 2-way-stop (2WSC) intersections. However, the previous conflict theory is extended in order to cope with the actual priority behavior at roundabouts. In addition, these systems are interacting according to the theory of chains of queues where the distance (= storage area) between the conflict points becomes important.

The paper explains the sophistication of the model, presents the mathematical derivations of ready-to-use capacity equations plus parameter calibration by existing data, and demonstrates real-world application. The advantages of the technique are: all conflicts - both between vehicle streams and pedestrian conflicts at entries and exits - are treated by the same congruent methods. Also the interaction between consecutive arms of the roundabout is modeled. By the technique it becomes also possible to model limited priorities (e.g. for pedestrians at crosswalks or zebra crossings) for all conflict points. Finally the conflicts which are decisive for the performance of the whole intersection are identified. The methods can easily be implemented into relevant computer software.

Keywords:

Roundabout capacity, conflict technique, quality of flow

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1 Introduction

The capacity of roundabouts is a matter of intense scientific investigations in many countries since the implementation of modern roundabouts in the 1980ies. The remarkable aspect is that each country has attempted to find its own solution. Up to some degree this may be justified by different driver attitudes or divergent traffic rules. Finally the differences between solutions in the various countries, however, have become rather small and may be mainly based on limited sample sizes or differing methods for analysis.

What, however, is common to each of the classical approaches is the fact that each entry to a roundabout is treated like an isolated T-junction. Effects of mutual interaction between the various conflict points of the roundabouts remain disregarded.

As one theory-based solution the maximum throughput (= capacity C) of one entry into the roundabout is calculated by the theory of unsignalized intersections. The easy way would be to apply the Siegloch-formula (Siegloch, 1973):

$$C = \frac{3600}{t_f} e^{-\frac{q_c}{3600} \cdot t_0} \tag{1}$$

with

= capacity	[veh/h]
= so-called zero-gap	[s]
$= t_c - t_f / 2$	
= follow-up time	[s]
= critical gap	[s]
= traffic volume on the circular lane in front of the entry	[veh/h]
	 capacity so-called zero-gap t_c - t_f / 2 follow-up time critical gap traffic volume on the circular lane in front of the entry

A consequent application would require reliable estimates for t_c and t_f (cf. Brilon, 2016). The other problem of this formula is that it is derived under the assumption of Poisson-distributed traffic on the circular lane (i.e. no bunching) which is more unrealistic at roundabouts than it is at Two-Way-Stop-Control (TWSC) intersections. Also effects of priority reversal or typical influences of the degree of saturation on t_c and t_f cannot be described.

As a consequence most researchers favor the so-called "empirical regression" approach for roundabout capacity estimation. Here the real-world traffic at a roundabout entry is counted over periods of uninterrupted queuing on the entry during specific time intervals, e.g. 1 minute. In case of a steady queue on the entry lane the observed flow volume is the capacity. This can be plotted over the observed conflict volume q_C on the circular lane. Result from this kind of empirical investigation is a cloud of points which shows that the capacity has a strong tendency to be reduced with increasing circular flow. Then a regression line is applied to represent these results.

The earliest attempt following this idea was the well-known British investigation by Kimber (1980). Here a linear regression equation for *C* as a function of q_C was applied where the parameters of the equation were modified according to geometric features of the intersection. Linear equations were also applied elsewhere, e.g. Brilon, Bondzio, and Wu (1997). Other authors found nonlinearities within their empirical results. They used exponential regression functions. One such solution is to use t_c and t_f in eq. (1) as regression parameters (e.g. Brilon and Wu, 2008). Others (e.g. Rodegerds e.a., 2015) transformed this into a simple exponential function of the type

$$C = A e^{-Bq_c} \tag{2}$$

with

С	= capacity	[veh/h]
A, B	= regression parameters	[-]
q_C	= traffic volume on the circular lane in front of the entry	[veh/h]

In this form the capacity formulas of the new HCM (2016) have become a standard for the US. Both forms of the regression analysis are equivalent.

The linear and the exponential regression approach for estimating the capacity at roundabouts are rather pragmatic. On one side it does not make use of theories for unsignalized intersections. On the other side one cannot be sure that these linear or exponential functions do also apply in areas of the C- q_C -diagram where only few measurement points have been observed. Therefore, Wu (1997) proposed the following formula for the capacity of a single lane entry to a roundabout:

$$C = \left(1 - \frac{q_C \tau}{3600}\right) \cdot \frac{3600}{t_f} \cdot \exp\left(-\frac{q_C}{3600} \cdot (t_0 - \tau)\right)$$
 [veh/h] (3)

where

С	= capacity of the entry	[veh/h]
q_C	= flow on circular lanes at the subject entry	[veh/h]
τ	= minimum headway between vehicles on the circular lane	[s]

Eq. (3) is the generalized form for the exponential and the linear capacity function. For example, this formula converges to an exponential function by $\tau = 0$ and to a linear function by $t_0 = \tau$.

There are also reports which found an influence of the exiting traffic on the entry capacity like Bovy e.a. (1991) for single lane roundabouts in Switzerland, Wu (2006) for double lane roundabouts with single lane exists, or Schmotz (2014) for mini roundabouts which found its way into the German guideline HBS (2015).

On the entries there may be another type of conflict which requires capacity consideration. Besides the conflicting flow on the circular lane this is the pedestrian crossing where pedestrians may be prioritized over the entering motorized vehicles, e.g. by a zebra crossing. The capacity reducing effect of the pedestrian crossing on the entry capacity has been studied by Stuwe (1992). Her results, which were based on a rather limited sample size of German zebra crossings, have been implemented into the German guidelines HBS (2001 and 2015) and also into the HCM (2010 and 2016).

Also the exits from the roundabout can establish a bottleneck for vehicle traffic with limited capacity which can also be influenced by pedestrians crossing the exit. A solution to describe this capacity has been presented by Marlow, Maycock (1982) in conjunction with Griffith's (1981) capacity formula for vehicular traffic traversing a zebra crossing. Even if the concept has been developed for junction entries it could also be applied for roundabout exits. Another more recent result for exit capacity at mini-roundabouts was presented by Schmotz (2014) – a solution which has the potential to be also applied for single-lane roundabouts.

Of course, the cited literature covers only a small extract from the large set of relevant publications within the international arena.

One commonality of all the results published up to now is that they treat specific conflicts at the roundabout with differing and incoherent methods and that they are not regarding any interaction between adjacent conflict areas. Under this aspect the solutions may be realistic as long as each element of the intersection is not operating near or above capacity. As soon as one of the conflict points is overloaded for a short period queues may be formed. Due to limited storage spaces on the circular lane these queues are impeding other conflict points with the consequence of reduced capacities there. Thus, a short overload at one point can easily become the starting point for an overload of the whole intersection – even if the analysis for all the single conflicts still leads to a sufficient service quality. This kind of mutual interdependencies of different parts of the roundabout are not confined to the case of a complete overload. Instead they occur already at a moderate increase of traffic demand. Due to this aspect it is desirable to formulate algorithms which describe the capacity of the roundabout as a whole regarding capacities of the various conflicts as well as the influence of limited internal storage areas. This paper tries to point out a mathematical model which is capable to estimate the capacity of the roundabout as a whole. The model represents the various conflict points by a homogeneous set of models including also a stochastic approach to cope with the interlocking between subsequent bottlenecks within the intersection.

2 Methodology

2.1 Capacity of traffic streams with priority control

As a conflict we treat the intersection of several streams which have to pass the same area within an intersection. The vehicles involved into a conflict have to pass the area one after the other. The set of streams which are involved into the same conflict is called a conflict group. The simplest case is a conflict group of two streams (Figure 1a). One of these streams (i) is assumed to have priority over the other (j).



Figure 1 - Conflict points between two streams (stream groups)

It can be assumed that any major stream vehicle passing a conflict point will occupy the conflict point for a certain time. Only during the unoccupied time the minor stream can pass the conflict point with a basic capacity C_0 . The basic capacity of a minor stream is the reciprocal of the follow-up time t_f . The conflict point can be occupied by a major stream vehicle in three different ways: it is occupied if there is a queue, a platoon, or a single arriving vehicle. A minor stream vehicle can only pass the conflict point if the conflict point is neither occupied by a queue nor by a platoon nor by a single arriving vehicle in the major stream (Wu, 2001). Thus, for a two-stream problem (Figure 1a), the capacity of a minor stream j entering a major stream i can be calculated with a very general model:

$$C_j = C_{0,j} p_{0,q,i} p_{0,p,i} p_{0,v,i}$$
 [veh/h] (4)

with

C_j	=	capacity of the minor stream <i>j</i>	[veh/h]	
$C_{0,j}$	=	basic capacity of the minor stream <i>j</i> in case of		
		no major stream (i) vehicle is occupying the conflict point		
	=	$3600/t_{f,j}$	[veh/h]	(5)
$p_{0,q,i}$	=	probability of queue-free state in the major stream <i>i</i>		

	=	impedance factor within the major stream $1 - x_i$ for queueing at the conflict point	[-]	(6)
$p_{0,p,i}$	=	probability of no vehicle platooning (vehicles following consecutively each other with a minimum headway τ_i) in the major stream <i>i</i>		
	=	$1 - \frac{q_{C,i}\tau_i}{3600}$	[-]	(7)
$p_{0,v,i}$	=	probability of no impedance caused by an upstream arriving	,	
		vehicle in the major stream <i>i</i>		
	=	$\Pr(\text{headway} > t_{0,ii})$		
	=	$\exp\left(-\frac{q_{C,i}}{3600}\cdot(t_{0,ij}-\tau_i)\right)$	[-]	(8)
t _{f i}	=	follow-up time of the minor stream <i>i</i>	[s]	
ac i	=	volume of the major stream		
10,1		(= circular stream at a roundabout) <i>i</i>	[veh/h]	
t _{0 ii}	=	$t_{ci} - t_{fi}/2$	[s]	(9)
t_{ci}	=	critical gap of the major stream <i>i</i> (Note, the critical gap is co	onsidered as	S
0,1		major stream related because the critical time is actually the	time consu	imed

c,*i* = critical gap of the major stream *i* (Note, the critical gap is considered as major stream related because the critical time is actually the time consumed by an arriving major stream vehicle at the conflict point. This is a more general consideration. Thus, in case of more than one major stream opposing a minor stream, this minor stream may have different values of the critical time.)

$$\tau_i$$
 = minimum time headway of the major stream *i* [s]
 x_i = degree of saturation of the major stream *i* [-]

For more than one major stream on the circular roadway (Figure 1b), the probabilities $p_{0,q}$, $p_{0,p}$, and $p_{0,v}$ for different major streams can be considered as independent of each other. Thus, for an minor stream *j* with multiple major streams *i* we get (Wu, 2001)

$$C_{j} = C_{0,j} \prod_{i \in I} p_{0,q,i} \prod_{i \in I} p_{0,p,i} \prod_{i \in I} p_{0,v,i}$$
[veh/h] (10)

where *I* is the set of all major streams under consideration.

The probability of a queue-free state $p_{0,q,i}$ is also known as the impedance factor f_{imp} . The impedance can also be caused by the queue from a downstream conflict point. The downstream queue and its impedance will be considered later in section 5. In this section, the queues in the major streams are neglected ($x_i = 0$) as a first approach. Thus $p_{0,q,i} = 1$ and therefore,

$$C_{j} = C_{0,j} \prod_{i \in I} p_{0,p,i} \prod_{i \in I} p_{0,v,i}$$
 [veh/h] (11)

That is,

with

$$C_{j} = C_{0,j} \prod_{i \in I} \exp\left(-\frac{q_{C,i}}{3600} \cdot (t_{0,ij} - \tau_{i})\right) \prod_{i \in I} \left(1 - \frac{q_{C,i}\tau_{i}}{3600}\right)$$

$$= C_{0,j} \exp\left(-\sum_{i \in I} \frac{q_{C,i}}{3600} \cdot (t_{0,ij} - \tau_{i})\right) \prod_{i \in I} \left(1 - \frac{q_{C,i}\tau_{i}}{3600}\right)$$
 [veh/h] (12)

All parameters mentioned in the model are related either to the major streams *i* or to the minor stream *j* or to both of them. This can be clearly seen regarding the indices *i* and *j*. That is, the critical gap $t_{c,i}$ and the minimum headway τ_i are related to the major stream *i*, the basic capacity $C_{0,j}$ is related to the minor stream *j*.

As a further simplification, the impedance caused by a single upstream arriving vehicle in the major stream can be neglected. That is we assume here $p_{0,v,i} = 1$ and thus

$$C_{j} = C_{0,j} \prod_{i \in I} \left(1 - \frac{q_{C,i} \tau_{i}}{3600} \right)$$
 [veh/h] (13)

The assumption that $p_{0,v,i} = 1$ corresponds to the condition $t_{0,ij} \approx \tau_i$. It means that the so-called zero-gap is of the same magnitude as the minimum time gap between major stream vehicles which can be considered as realistic in most of cases for traffic behavior at a roundabout. Eq. (13) is a linear function for a single lane circular roadways and a quadratic, convex function for double lane circular roadways. Both shapes of capacity functions can be observed in reality respectively.

In case of a "limited priority" (Troutbeck and Kako, 1997, major stream *i* yielding to minor stream *j* to some specific extend) this equation can be expressed as following (cf. also Miltner, 2003).

$$C_{j} = C_{0,j} \prod_{i \in I} \left(1 - b_{ij} \frac{q_{C,i} \tau_{i}}{3600} \right)$$
 [veh/h] (14)

with

$$b_{ij}$$
 = probability of right of way for a major stream
on the circular lane (b_{ij} =1: no *i* yielding to *j*,
or in other words: the priority of *i* is always obeyed by *j*) [-]

The values of b_{ij} can be defined according to a so-called conflict matrix. The probability of right of way b_{ij} is related to both of the major stream *i* and the minor stream *j*.

The total capacity of an entry with more than one lane is the sum of capacities of all entry lanes (Figure 2a). That is,

$$C = \sum_{j \in J} C_j = \sum_{j \in J} \left(C_{0,j} \prod_{i \in I} \left(1 - b_{ij} \frac{q_{C,i} \tau_i}{3600} \right) \right)$$
[veh/h] (15)

where J is the set of all minor streams under consideration.



Figure 2 - Conflict points with more than one minor stream

For estimating the total entry capacity, the proportion of the volumes on each entry lane is assumed to remain constant, and the total entry capacity is subject to some boundary condition. For example, for a double lane entry the following condition has to be satisfied:

$$C = \min\left(C_{2L}\frac{q_{2L} + q_{2R}}{q_{2L}}; C_{2R}\frac{q_{2L} + q_{2R}}{q_{2R}}\right) = \min\left(\frac{C_{2L}}{a_{2L}}; \frac{C_{2R}}{a_{2R}}\right)$$
 [veh/h] (16)

with

$$a_{2L} = q_{2L} / (q_{2L} + q_{2R})$$

= proportion of traffic in the entry for lane 2L [-]
$$a_{2R} = q_{2R} / (q_{2L} + q_{2R})$$

= proportion of traffic in the entry for lane 2R [-]

That is the maximum capacity of the two lanes together. Eq. (16) shows that the maximum capacity of a double lane entry is a function of the distribution of traffic volumes on both lanes. For example, if lane 2*R* accommodates 80% of the total entry capacity, say $a_{2R} = 0.8$, and the capacity of the lane 2*R* is 600 veh/h, the maximum capacity of the double lane entry is than 600/0.8 = 750 veh/h. That is much less than the two-fold of the single lane capacity (600*2 = 1200 veh/h).

2.2 Multilane entry with limited length of lanes

If one of the entry lanes has only a limited length n_s , and both lanes emerge from a single upstream lane (Figure 2b), the capacities of the single lanes at the entry cannot be utilized as calculated. According to the usual concept of unsignalized intersections, we have here a "short-share lane" problem. The capacity of shared lanes can be determined according to a formula first developed by Harders (1968). This concept has been extended by Wu (1997) such that also additional lanes of limited length (short lane) can be taken into account. For the case of a single-lane approach with an additional short lane near the intersection the capacity of the entry with short-share traffic lane can be calculated from (Wu, 1997):

$$C_{s} = \frac{q_{2L} + q_{2R}}{\sqrt[n_{s}+1]{(x_{2L})^{n_{s}+1} + (x_{2R})^{n_{s}+1}}} = \frac{q_{2L} + q_{2R}}{\sqrt[n_{s}+1]{\left(\frac{q_{2L}}{C_{2L}}\right)^{n_{s}+1} + \left(\frac{q_{2R}}{C_{2R}}\right)^{n_{s}+1}}}$$
[veh/h] (17)

with

C_s	=	capacity of the short-shared lane	[veh/h]
q_{2L}	=	volume for the entry lane 2L	[veh/h]
q_{2R}	=	volume for the entry lane 2 <i>R</i>	[veh/h]
C_{2L}	=	capacity for the entry lane 2L	[veh/h]
C_{2R}	=	capacity for the entry lane $2R$	[veh/h]
x_{2L}	=	degree of saturation of the entry lane 2L	[-]
x_{2R}	=	degree of saturation of the entry lane $2R$	[-]

In case n_s goes to infinite, eq. (17) yields eq. (16).

For $n_s = 0$, eq. (17) yields the well-known shared lane formula:

$$C_{s} = \frac{q_{2L} + q_{2R}}{x_{2L} + x_{2R}} = \frac{q_{2L} + q_{2R}}{\frac{q_{2L}}{C_{2L}} + \frac{q_{2R}}{C_{2R}}}$$
[veh/h] (18)

2.3 The two-stage queuing problem

Eq. (14) deals only with cases where the major streams are crossed by a minor stream at an isolated conflict point. In case of a pedestrian crossing at a roundabout, there are often several spaces between the circular major stream and the pedestrian crossing (Figure 3). The minor stream at the entry can pass the pedestrian crossing and the circular major stream one-by-one and if necessary wait in between. The capacity of this two-stage queuing system with n_w waiting places is given by Brilon and Wu (1999).



Figure 3 - Two-stage priority with n_w waiting places

We consider two consecutive queuing stages a and b with n_w possible waiting places in between. Then the total capacity of the combined queuing system can be expressed by

$$C_T = \begin{cases} \frac{1}{n_w + 1} \left(n_w C_b + C_{ab} \right) & \text{for } y = 1 \\ \left(1 - w_0 \right) C_b + w_0 C_{ab} & \text{elsewhere} \end{cases}$$

$$(19)$$

where

= total capacity of the combined queuing system C_T $= \frac{y-1}{v^{n_w+1}-1}$ w_0 = possible waiting places between both queueing stages n_w C_a $= C_{0,a} p_{0,a}$ = capacity of the first stage a C_{h} $= C_{0,b} p_{0,b}$ = capacity of the second stage b C_{ab} $= C_{0,ab} p_{0,a} p_{0,b}$ = capacity of queuing system in case $n_w = 0$ $= \frac{C_a - C_{ab}}{C_b - C_{ab}}$ y

Different applications for the two-stage model are described in the following chapters. The calculation of C_a and C_b is explained for each case in these chapters.

In Brilon and Wu (1999), the capacity C_{ab} of the queuing system in case of $n_w = 0$ is only given for a special case with $C_{0,a} = C_{0,b} = C_{0,ab}$. For the two-stage problem at roundabouts regarding pedestrians this assumption does not apply. However, also in the general case with $C_{0,a} \neq C_{0,b} \neq C_{0,ab}$ the capacity C_{ab} can be calculated. The capacity C_{ab} of the queuing system in case of $n_w = 0$ can be expressed as a function of capacities of the two stages. That is,

$$C_{ab} = C_{0,ab} p_{0,a} p_{0,b} = C_{0,ab} \frac{C_a}{C_{0,a}} \frac{C_b}{C_{0,b}} = f_{ab} C_a C_b$$
 [veh/h] (20)

with

$$f_{ab} = \frac{C_{0,ab}}{C_{0,a}C_{0,b}}$$

This leads to

$$y = \frac{C_a - C_{ab}}{C_b - C_{ab}} = \frac{C_a (1 - f_{ab} C_b)}{C_b (1 - f_{ab} C_a)}$$
[-] (21)

The values of C_0 are actually the reciprocal of the follow-up time t_f . It is the time headway h at lane capacity C_{\ln} plus the lost time Δt experienced by an approaching vehicle needed for deceleration and orientation maneuver. That is

$$\Delta t = t_f - h = \frac{3600}{C_0} - \frac{3600}{C_{\ln}}$$
[s] (22)

For crossing stage *a* and *b* in one step, the deceleration and orientation time Δt is the sum of Δt in both stages. That is,

$$\Delta t_{ab} = \Delta t_a + \Delta t_b = \frac{3600}{C_{0,a}} + \frac{3600}{C_{0,b}} - \frac{2 \cdot 3600}{C_{ln}}$$
[s] (23)

Thus,

$$C_{0,ab} = \frac{3600}{h + (\Delta t_a + \Delta t_b)} = \frac{1}{\frac{1}{C_{0,a}} + \frac{1}{C_{0,b}} - \frac{1}{C_{\ln}}}$$
[veh/h] (24)

In the following, a time headway of h = 2.2 s is used at roundabouts. Thus, the lane capacity C_{ln} is 3600/2.2s = 1640 veh/h.

Eq. (19) was derived for a queuing system with Markovian arrivals and departures. In the reality those presumptions are rarely satisfied. For accounting for the stochastic property of a queuing system, a factor C_n can be applied to the parameter n_w . That is, as an approximation to the stochastic property of the queuing system instead of n_w a parameter $n_w^* = C_n n_w$ can be used in eq. (19). For a queuing system with Markovian arrivals and departures is $C_n = 1$. The factor C_n is normally greater than 1. For example, one can use $C_n = 1.68$ for a queuing system with Markovian arrivals and deterministic service times (Wu, 1994). The value of C_n is subject to calibration.

In general, the total capacity of a two-stage queuing system can be expressed with 4 significant parameters (cf. eq. (19)):

$$C_T = f(C_a, C_b, C_{0,ab}, n_w^*)$$
 [veh/h] (25)

The formulation (eq. (19)) of the capacity of a two-stage queuing system is very complex. As a simplification the following formulation can be used instead of eq. (19):

$$C_{T} = \min \begin{cases} C_{a} \left(1 - \left(1 - f_{ab} C_{b} \right)^{n_{w}^{*} + 1} \right) \\ C_{b} \left(1 - \left(1 - f_{ab} C_{a} \right)^{n_{w}^{*} + 1} \right) \end{cases} = f(C_{a}, C_{b}, C_{0, ab}, n_{w}^{*}) \qquad [\text{veh/h}] \quad (26)$$

For practical applications this approximation provides a rather good fit.

3 Conflict groups at a roundabout

All capacities of the individual conflicts or conflict groups at a roundabout can be estimated using the equations presented in section 2. The total set of conflict points at a roundabout with 4 arms and 12 movements is illustrated in Figure 4. For a particular double lane entry yielding to a double lane circular roadway, the traffic streams and the involved conflict points are depicted in Figure 5. The traffic lanes are named left (L) and right (R) entry lane, outer (O), inner (I) circular lane, and pedestrian crosswalk (PE).



Figure 4 – Conflict points at roundabouts in the real world

In order to calculate the entry capacity, the lane volume distributions at the entry and on the circular roadway have to be estimated in advance according the *O*-*D* relationship of all movements at the intersection.

From the *O*-*D* relationship the circular volume q_c , the entry volume q_E , and the exit volume q_A at the subject approach *k* can be calculated as follows.

$$q_{C,k} = q_{k-1,k+1} + q_{k-1,k+2} + q_{k-2,k+1} + (q_{k-3,k+1})$$
 [veh/h] (27)

$$q_{E,k} = q_{k,k+1} + q_{k,k+2} + q_{k,k+3}$$
 [veh/h] (28)

$$q_{A,k} = q_{k+1,k} + q_{k+2,k} + q_{k+3,k}$$
 [veh/h] (29)

with k = number of the arm. $q_{k,m}$ is the volume from arm k to arm m (cf. Figure 4). In case that m > 4 it has to be transformed into m - 4. For a roundabout with 3 or more than 4 arms the calculations have to be adequately adjusted. The volume in parentheses is the U-turn volume which does not always exist.

For double lane roundabouts, one can assume that the volume on the left entry lane (L) corresponds to the left-turn volume at the entry. Furthermore, it can be assumed that nearly none (0%) of the left-turn vehicle from the upstream arm will use the inner circular lane (I) at a very low circular volume and nearly all of them will use the inner circular lane at high circular volume (up to 1600 veh/h). In between the volumes can be obtained by interpolation. These assumptions are realistic because the inner circular lane and the left entry lane are mostly used by the corresponding left-turn vehicles. That is,

$$q_{C,I,k} = q_{L,k-1} \frac{q_C}{1600} = q_{k-1,k+2} \frac{q_C}{1600}$$
 and $q_{L,k} = q_{k,k+3}$ [veh/h] (30)

For example, for an entry with two lanes (L and R) shown in Figure 5a with one outer (O), one inner (I) circular lane, and one pedestrian crosswalk (PE), the capacity of the left and the right entry are

$$C_{L} = C_{0,L} \left(1 - b_{IL} \frac{q_{c,I} \tau_{I}}{3600} \right) \left(1 - b_{OL} \frac{q_{c,O} \tau_{O}}{3600} \right) \left(1 - b_{PEL} \frac{q_{c,PE} \tau_{PE}}{3600} \right)$$
 [veh/h] (31)

$$C_{R} = C_{0,R} \left(1 - b_{OR} \frac{q_{c,O} \tau_{O}}{3600} \right) \left(1 - b_{PER} \frac{q_{c,PE} \tau_{PE}}{3600} \right)$$
 [veh/h] (32)



Figure 5 – An entry with two lanes in the entry and two circular lanes

In the real world, there is normally a waiting area with n_E storage places ($n_E \ge 1$) between the outer circular lane and the pedestrian crosswalk. A minor stream vehicle can cross the pedestrian crosswalk first and stay in between. In this case the capacity of a minor stream must be calculated for crossing the pedestrian crosswalk (C_a) and for entering the major stream (C_b). Thus,

$$C_{L,a} = C_{0,PEL} \left(1 - b_{PEL} \frac{q_{C,PE} \tau_{PE}}{3600} \right)$$
 [veh/h] (33)

$$C_{L,b} = C_{0,L} \left(1 - b_{IL} \frac{q_{C,I} \tau_I}{3600} \right) \left(1 - b_{OL} \frac{q_{C,O} \tau_O}{3600} \right)$$
 [veh/h] (34)

The combined capacity of this two-stage problem can be obtained from eq. (19) or eq. (26):

$$C_{L,T} = f(C_{L,a}, C_{L,b}, C_{0,L,ab}, n_E)$$
 [veh/h] (35)

Similarly for the right entry lane we have

$$C_{R,a} = C_{0,PER} \left(1 - b_{PER} \frac{q_{c,PE} \tau_{PE}}{3600} \right)$$
 [veh/h] (36)

$$C_{R,b} = C_{0,R} \left(1 - b_{OR} \frac{q_{c,O} \tau_O}{3600} \right)$$
 [veh/h] (37)

$$C_{R,T} = f(C_{R,a}, C_{R,b}, C_{0,R,ab}, n_E)$$
 [veh/h] (38)

where f is the functionality given by eq. (19) or eq. (26).

Obviously, for a multi-lane roundabout, the volume distribution between q_I and q_O on the circular lanes and the volume distribution between q_L and q_R on the entry lanes must be calculated in advance. The distributions of those volumes can be estimated according to the turning movements at the intersection. For a single lane roundabout, for all equations the indices *I* and *L* are no longer applicable and they just must be neglected.

Taking into account the volumes of both left (q_L) and right (q_R) entry lanes and the length of the double-lane area (n_d) upstream from the pedestrian crossing, the total capacity of the double-lane entry $(C_{E,d})$ can then be calculated using eq.(17).

$$C_{E,d} = \frac{q_L + q_R}{\binom{n_d + 1}{C_{L,T}}^{n_d + 1} + \left(\frac{q_R}{C_{R,T}}\right)^{n_d + 1}}$$
[veh/h] (39)

Setting $n_d = 0$, this equation yields the shared lane capacity ($C_{E,s}$) of a single lane entry opposing two circular lanes at a roundabout:

$$C_{E,s} = \frac{q_L + q_R}{\frac{q_L}{C_{L,T}} + \frac{q_R}{C_{R,T}}}$$
 [veh/h] (40)

In this case two minor streams on one entry lane are actually calculated (cf. Figure 5a). One of them (q_L) crosses the outer circular lane and proceeds into the inner circular lane. And another (q_R) continues directly into the inner circular lane.

Note, at a single lane entry the approach arm has normally a flare area to enable the turning movement of the right-turn vehicle entering the roundabout. Thus, there is actually a double lane area able to accommodate one vehicle. Under real world conditions, this flare area is not used by all vehicles but only by a portion a_f of them. Thus the capacity of the single lane entry yielding to a double lane circular roadway is

$$C_{E,\text{single lane}} = a_f C_{E,d} (n_d = 1) + (1 - a_f) C_{E,s}$$

= $a_f \frac{q_L + q_R}{\sqrt{\left(\frac{q_L}{C_{L,T}}\right)^2 + \left(\frac{q_R}{C_{R,T}}\right)^2}} + (1 - a_f) \frac{q_L + q_R}{\frac{q_L}{C_{L,T}} + \frac{q_R}{C_{R,T}}}$ [veh/h] (41)

Again, the value of a_f is subject to calibration depending on the geometric layout of the entry area. With $a_f = 0$ one is on the safer side of calculation.

Setting the traffic volume on the inner circular lane $q_I = 0$, all formulas mentioned here can be used for a roundabout with a single lane circular roadway.

As a summary, all parameters mentioned above can be defined using a conflict matrix. Table 1 through Table 4 show the calibrated parameters representing German roundabouts based on methods and data from the HBS (2015). The basic capacities of the minor stream $C_{0,j}$ and the minimum headway τ_i of the corresponding major stream mentioned above are given for traffic streams under consideration (veh, ped, and two-stage). For the calibration of double lane roundabouts, it was assumed that nearly no (0%) circular vehicles are using the inner circular lane at very low circular volume and 30% at high circular volume (1600 veh/h). For a double lane entry 30% of the total entry volume is assumed to use the left entry lane. These assumptions are realistic because the inner circular lane and the left entry lane are mostly used by the corresponding left-turn vehicles and due to the fact that in Germany the left lanes at a two-lane roundabout are used by the drivers rather reluctantly. That is,

$$\frac{q_{C,I}}{q_C} = \frac{0.3q_C}{1600} \text{ or } q_{C,I} = \frac{0.3q_C^2}{1600}$$
 [veh/h] (42)

and

$$q_L = 0.3q_E \qquad [\text{veh/h}] \quad (43)$$

Table 1 – Conflict matrix for parameters of a roundabout entry with or	ne entry lane and
two circular lanes (1/2)	

Major Minor lane j	Circular inner (<i>I</i>)	Circular outer (<i>O</i>)	Ped (PE)	$C_{0,j}$ [veh/h]			Places btw. two	
		Stage b		Stage a				stages n_E
	$\tau_i[s]$	2.3	2.3	2.3	Veh Stage b	Ped Stage <i>a</i>	2-Stage <i>ab</i>	[veh]
Entry left (<i>L</i>)	b_{ij}	0.9	0.9	0.9	1270	1330	1080	1
Entry right (<i>R</i>)	b_{ij}	0.9	0.9	0.9	1420	1420	1250	1

Table 2 – Conflict matrix for parameters of a roundabout entry with two entry lanes and two circular lanes (2/2)

Major lane <i>i</i> Minor lane <i>j</i>		Circular inner (I)	Circular outer (<i>O</i>)	Ped (PE)	$C_{0,j}$ [veh/h]			Places btw. two
		Stage <i>b</i>		Stage a				n_E
	$\tau_i[s]$	2.4	2.4	2.6	Veh Stage <i>b</i>	Ped Stage <i>a</i>	2-Stage <i>ab</i>	[veh]
Entry left (<i>L</i>)	b_{ij}	0.9	0.9	0.9	1010	1010	730	1
Entry right (<i>R</i>)	b_{ij}	0.9	0.9	0.9	1100	1100	830	1

Table 3 – Conflict matrix for parameters of a roundabout entry with single lane entry and single circular lane (1/1)

Major lane		Circular lane	Ped (PE)	$C_{0,j}$ [veh/h]			Places btw. two	
y		Stage b	Stage a					
	$\tau_i[s]$	1.8+14.5/D ^{*)}	2.8	Veh Stage <i>b</i>	Ped Stage <i>a</i>	2-Stage <i>ab</i>	n_E [veh]	
Entry lane	b_{ij}	0.9	0.9	1200	1550	1150	1	

*) D = outer diameter of the roundabout [m]

Table 4 – Conflict matrix for parameters of mini roundabout entry with single lane entry and single circular lane (1/1 mini)

Major lane		Circular lane	Ped (PE)	$C_{0,j}$ [veh/h]			Places btw. two
y		Stage b	Stage a				
	$\tau_i[s]$	2.6	3.0	Veh Stage b	Ped Stage <i>a</i>	2-Stage Ab	n_E [veh]
Entry lane	b_{ij}	0.9	0.9	1080	1500	1020	1



Figure 6 – Comparison of the model calibration to the HBS 2015 data dots=data from HBS 2105, lines=models



Figure 7 – Comparison of the model calibration to the HBS 2015 data for impedance factor of pedestrians, dots=data from HBS 2105, lines=models

In Figure 6, a comparison of the model calibration to the HBS 2015 data is depicted. It can be seen, that the simplified model can be calibrated to the HBS data very well. Figure 7 shows a comparison of the model to the HBS 2015 data with regards to the pedestrian impedance

factor at roundabout entries. For single lane roundabouts the model results represent the HBS 2015 methodology very well. For double lane roundabouts, the results of HBS 2015 cannot represent the whole range of the circular volume. The reason of those deviations must be found in the HBS 2015 model, because the HBS 2105 model is a regression based on limited data without any theoretical background. However, in the common range of circular volume the results of HBS 2015 data can also be represented by the new model properly.

4 Capacity of an exit at a roundabout

An exit at a roundabout can also be considered as a two-stage queueing system (cf. Figure 8)



Figure 8 – An exit with two circular lanes

The combined capacity of this two-stage problem can be obtained from eq. (19) or eq. (26):

$$C_{A,T} = f(C_{A,a}, C_{A,b}, C_{0,A,ab}, n_A)$$
 [veh/h] (44)

where $C_{A,a} = C_{\ln}$ is the capacity of the exit lane at the edge of the outer circular lane. $C_{A,b}$ is the capacity of the pedestrian crossing at the exit, and $C_{0,A,ab}$ is the basic capacity of the two-stage problem at the exit.

The capacity $C_{A,b}$ can be calculated as

$$C_{A,b} = C_{A,PA} = C_{0,RAR} \left(1 - b_{PAR} \frac{q_{a,PA} \tau_{PA}}{3600} \right)$$
 [veh/h] (45)

The upstream outer circular lane at an exit can be treated as a shared lane consisting of the exit lane (q_A) and the downstream outer circular lane $(q_{O,D})$. Thus, the capacity of the upstream outer circular lane at an exit is

$$C_{O,U} = \frac{q_A + q_{O,D}}{\frac{q_A}{C_{A,T}} + \frac{q_{O,D}}{C_{O,D}}}$$
 [veh/h] (46)

In accordance, the parameters for an exit are given in Table 5. The lane capacity of the downstream outer circular lane $C_{O,D}$ is set to the lane capacity C_{\ln} (1640 veh/h).

Major lane <i>i</i> Minor lane <i>j</i>		Exit lane at edge of the outer circular lane with $C_j = C_{0,j}$	Ped (PA)	$C_{0,j}$ [veh/h]			Places btw. two	
		Stage a	Stage b				stages n_E	
	$ au_i[\mathbf{s}]$	-	2.9	Veh Stage <i>a</i>	Ped Stage b	2-Stage <i>ab</i>	[veh]	
Exit lane (A)	b_{ij}	1	0.9	1400	1550	1330	1	
Upstream outer circular lane as a share lane (O, U)		Exit lane (A)					$C_{A,T}$	
		Downstream outer circular (O,D)					1640	

Table 5 - Conflict matrix for parameters of a roundabout exit with only one exit lane

5 Reduction of entry capacity at a roundabout due to queuing at the downstream exit



Figure 9 - An entry - exit constellation with two lanes in the entry and two circular lanes

For both the left and right entry lane, the capacity can be impeded by queuing vehicles on the outer circular lane caused by the downstream exit. Assuming that the downstream exit obeys an M/G/1 queuing system, the impedance factor $f_{imp,EA}$ can approximately be calculated as following.

$$f_{imp,EA} = p_{0,EA,n_{EA}} = 1 - x_{O,EA}^{C_n n_{EA} + 1}$$
[-] (47)

with

 $p_{0,EA,n_{EA}}$ = probability that the entry is not impeded by queuing from the downstream exit = probability that the space between the downstream exit and the subject entry is not totally occupied by the downstream queue $= \Pr(Q_{O,EA} \le n_{EA})$ $= 1 - x_{O,EA}^{C_n n_{EA} + 1}$ [-] = degree of saturation of the downstream outer circular lane $\chi_{O.EA}$ $= q_{O,EA}/C_{O,EA}$ [-] = volume of the downstream outer circular lane [veh/h] $q_{O,EA}$ = capacity of the downstream outer circular lane [veh/h] $C_{O,EA}$ = storage places between the entry and the downstream n_{EA} exit on the outer circular lane [veh]

C_n = Factor accounting for the stochastic property of the queuing system.

In this equation, the service time of the queuing system is considered less stochastic using a factor $C_n = 1.68$ applied to the parameter n_{EA} assuming the service time of the queuing system on a circular lane is nearly deterministic (Wu, 1994). In Table 6, the parameters for calculating the impedance caused by the downstream queue are given. The value of $n_{EA} = 3$ is assumed for a midsize single lane roundabout with an outer diameter D = 35m. For a real world roundabout, the value of n_{EA} can be obtained from the given geometry.

Major Minor lane <i>j</i>	lane i	Downstream outer lane at the entry with $C_j = C_{0,j}$	Upstream outer lane at the exit with $C_j = C_{O,U}$ from Eq.(46)		Places btw. two stages		
		Stage a	Stage b			[veh]	
	$\tau_i[s]$	-	-	Stage a	Stage b	2-Stage <i>ab</i>	
Outer circular lane btw. an entry and an exit (<i>EA</i>)	b_{ij}	1	1	1640	1640	1640	1.68*3

Table 6 – Conflict matrix for parameters of an entry – exit constellation

Considering the impedance of queuing caused by the downstream exit, the capacity of the second stage of the left lane is

$$C_{L,b}^{*} = \prod p_{0,q,i} C_{L,b} = p_{0,EA,n_{EA}} C_{L,b} = f_{imp,EA} C_{L,b}$$
 [veh/h] (48)

with $C_{L,b}$ form eq. (34) and $f_{imp,EA}$ from eq. (47).

The capacity of the first stage remains unchanged:

$$C_{L,a}^{*} = C_{L,a}$$

with $C_{L,a}$ from eq. (33). And with the functionality f from eq. (19) or eq. (26),

$$C_{L,T}^{*} = f(C_{L,a}^{*}, C_{L,b}^{*}, C_{0,L,ab}, n_E)$$
 [veh/h] (49)

Similarly, for the right entry lane we get

$$C_{R,b}^{*} = p_{0,EA,n_{EA}} C_{R,b} = f_{imp,EA} C_{R,b}$$
 [veh/h] (50)

$$C_{Ra}^{*} = C_{Ra} \qquad [\text{veh/h}] \quad (51)$$

with $C_{R,a}$ from eq. (36) and $C_{R,b}$ from eq. (37). Thus, with functionality f from eq. (19) or eq. (26),

$$C_{R,T}^{*} = f(C_{R,a}^{*}, C_{R,b}^{*}, C_{0,R,ab}, n_E)$$
 [veh/h] (52)

All parameters are defined previously.

For applications in the practice, the following steps of an algorithm with the corresponding equations are summarized as a guide for the calculation procedure at an entry-exit constellation:

[-]

- 1 Estimation of the demand volumes q_C , q_E and q_A at the subject entry and exit (eqs. (27), (28), and (29))
- 2 Estimation of the distributed demand volumes (if applicable) or lane demand volumes on the circular lanes q_I and q_O and on the entry lanes q_L and q_R according to applicable assumptions
- 3 Downstream exit:
 - 3.1 Estimation of the first stage capacity C_a of the exit lane at the edge of the outer circular lane (e.g. $C_a = 1400$ veh/h)
 - 3.2 Estimation of the second stage capacity C_b against the pedestrian stream (C_b from eq. (45))
 - 3.3 Estimation of the two-stage capacity $C_{A,T}$ at the exit lane ($C_{A,T}$ from eq. (44), C_a from point 3 and C_b from point 3.2)
 - 3.4 Estimation of the shared lane capacity $C_{O,U}$ of the diverge point at the exit (eq. (46)) consisting of the downstream circular lane ($C_{O,D} = C_{\ln}$) and the exit lane ($C_{T,A}$ from point 3.3)
 - 3.5 Estimation of the impedance factor $f_{imp,EA}$ (eq. (47)) to the upstream entry lanes due to queues from the downstream circular lane ($C_{O,U}$ from point 3.4)
- 4 Upstream entry (entry under consideration):
 - 4.1 Estimation of the first stage capacities C_a on entry lanes against the pedestrian stream (eqs. (33) and (36))
 - 4.2 Estimation of the second stage capacities C_b on entry lanes against the circular major stream (eqs. (34) and (37))
 - 4.3 Estimation of the second stage capacities C_a^* and C_b^* impeded by the downstream impedance factor (eqs. (48) and (50), $f_{imp,EA}$ from point 3.5)
 - 4.4 Estimation the two-stage capacities $C_{L,T}^*$ and $C_{R,T}^*$ at the entry lanes (eqs. (49) and (52))
 - 4.5 Estimation of the shared lane capacity $C_{E,d}$ (if applicable, eq. (39), with $C_{L,T} = C_{L,T}^*$ and $C_{R,T} = C_{R,T}^*$ from point 4.4)

6 Evaluation of the proposed model

To evaluate the proposed model, two examples are constructed for a single lane roundabout using the calibrated parameters in sections 3 through 5. One example is for an entry – exit constellation where the blockage effect due to queue spillback from the downstream exit is demonstrated. And another example deals with the whole roundabout considering all the entries and exits in a consecutive way.

For the evaluation, we consider a roundabout connecting a major (arms 1 and 3, cf. Figure 4) and a minor (arms 2 and 4) road. The flow split of the major and the minor volume is 60% to 40% of the total intersection volume. The volumes of the turning movements are defined correspondingly. That is, 20%/60%/20% at a major street arm and 30%/40%/30% at a minor street arm.

At first, an entry - exit constellation at a single-lane roundabout with all possible conflict points depicted in Figure 10 is considered. For this constellation, the conflict against the circular major stream (A), against the pedestrian stream (B) at the entry, and the combination of both (BA) are calculated. At the exit, the conflict against the pedestrian stream (D), the conflict at the edge of the circular lane (C), and the combination of both have to be dealt with. In addition, the capacity of the diverge point at the exit (F) is calculated as a shared lane consisting of the downstream circular lane (H) and the exit lane (CD). Furthermore the, impedance factor ($f_{imp,EF}$) due to queues from the downstream circular lane (F) at the exit to the upstream circular lane (E) at the entry is calculated. The capacity of the entry (A) and of

the up- and downstream circular lane (G and E) at the entry are reduced by the impedance factor.



Figure 10 – Conflict points of an entry-exit constellation

Due to the different entry and exit volumes at a major street and a minor street entry and due to the symmetric volumes there are two cases for the traffic volumes on the circle:

- A) major street entry followed by a minor street exit
- B) minor street entry followed by a major street exit

At first, the investigated roundabout is loaded by a total intersection volume of 1900 veh/h. The results of the evaluation are depicted in Table 7 for case A) and for case B) in Table 8. As a result the maximum degree of saturation at a major street arm is 0.663 and at a minor stream arm is 0.635.

Then, the total volume is raised step by step in order to investigate the total intersection capacity and the interaction between the entry and the downstream exit. For the major street arms (case A), the results of the evaluation are depicted in Table 9. The results for the minor street arms (case B) are shown in Table 10. According to the evaluation results, the example roundabout can at maximum accommodate a total intersection volume of 2297 veh/h. This corresponds to an ADT of ca. 28000 veh/d (for typical urban conditions of a peak hour volume = ADT * 0.08). At this volume (intersection capacity) the degree of saturation for conflict BA at the major street arms is nearly equal to 1 (0.999). The corresponding degree of saturation at a minor street arm is 0.854.

Point	q_i	$q_{C,i}$	$C_{0,j}$	$ au_i$	b_{ij}	п	<i>C</i> *	$f_{imp,EF}$	С	x	Note
А	570	380	1200	2.2	0.9	-	921	0.917	870	0.655	Entry
В	570	100	1550	2.8	0.9		1429	-	1429	0.399	Ped at entry
BA	570	-	-	-	-	1	938	-	869	0.656	2-stage, entry
С	570	-	1400	-	-	-	1400	-	1400	0.407	Exit
D	570	100	1550	2.8	0.9	-	1442	-	1442	0.395	Ped at exit
CD	570	-	-	-	-	1	1322	-	1322	0.431	2-stage,exit
Н	380	-	1640	-	-	-	1640	-	1640	0.232	Downstream of exit
											Shared lane H/CD,
F	950	-	-	-	-	-	1433	-	1433	0.663	upstream of exit
Е	950	-	1640	-	-	3	1640	0.917	1503	0.632	Downstream of entry
G	380		1640				1640	0.917	1503	0.253	Upstream of entry

Table 7 – Capacity of the conflict points of an entry - exit constellation (A): major street entry - minor street exit) with total intersection volume of 1900 veh/h

max x 0.663

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Point	q_i	$q_{C,i}$	$C_{0,j}$	$ au_i$	b_{ij}	n	<i>C</i> *	$f_{imp,EF}$	С	x	Note
А	380	570	1200	2.2	0.9	-	782	0.936	771	0.493	Entry
В	380	100	1550	2.8	0.9		1429	-	1429	0.266	Ped at entry
BA	380	-	-	-	-	1	814	-	770	0.493	2-stage, entry
С	380	-	1400	-	-	-	1400	-	1400	0.271	Exit
D	380	100	1550	2.8	0.9	-	1442	-	1442	0.264	Ped at exit
CD	380	-	-	-	-	1	1322	-	1322	0.287	2-stage,exit
Н	570	-	1640	-	-	-	1640	-	1640	0.348	Downstream of exit
F	950	-	-	-	-	-	1496	-	1496	0.635	Shared lane H/CD, upstream of exit
											Downstream of
Е	950	-	1640	-	-	3	1640	0.936	1534	0.619	entry
G	570		1640				1640	0.936	1534	0.371	Upstream of entry
										0.00	

Table 8 – Capacity of the conflict points of an entry - exit constellation(B): minor street entry - major street exit) with total intersection volume of 1900 veh/h

max x 0.635

Table 9 – Capacity of the conflict points of an entry - exit constellation (A): major street entry - minor street exit) with total intersection volume of 2297 veh/h

Point	q_i	$q_{C,i}$	$C_{0,j}$	$ au_i$	b_{ij}	п	<i>C</i> *	$f_{imp,EF}$	С	x	Note
А	689	459	1200	2.2	0.9	-	863	0.779	698	0.987	Entry
В	689	100	1550	2.8	0.9		1429	-	1429	0.482	Ped at entry
BA	689	-	-	-	-	1	886	-	690	0.999	2-stage, entry
С	689	-	1400	-	-	-	1400	-	1400	0.492	Exit
D	689	100	1550	2.8	0.9	-	1442	-	1442	0.478	Ped at exit
CD	689	-	-	-	-	1	1381	-	1381	0.499	2-stage,exit
Н	459	-	1640	-	-	-	1640	-	1640	0.280	Downstream of exit
											Shared lane H/CD,
F	1149	-	-	-	-	-	1474	-	1474	0.779	upstream of exit
Е	1149	-	1640	-	-	3	1640	0.779	1277	0.900	Downstream of entry
G	459		1640				1640	0.779	1277	0.359	Upstream of entry
										0.000	

max x 0.999

Table 10 – Capacity of the conflict points of an entry - exit constellation (B): minor street entry - major street exit) with total intersection volume of 2297 veh/h

Point	q_i	$q_{C,i}$	$C_{0,j}$	$ au_i$	b_{ij}	n	<i>C</i> *	$f_{imp,EF}$	С	x	Note
А	459	689	1200	2.2	0.9	-	695	0.820	611	0.751	Entry
В	459	100	1550	2.8	0.9		1429	-	1429	0.321	Ped at entry
BA	459	-	-	-	-	1	736	-	604	0.760	2-stage, entry
С	459	-	1400	-	-	-	1400	-	1400	0.328	Exit
D	459	100	1550	2.8	0.9	-	1442	-	1442	0.318	Ped at exit
CD	459	-	-	-	-	1	1381	-	1381	0.332	2-stage,exit
Н	689	-	1640	-	-	-	1640	-	1640	0.420	Downstream of exit
F	1149	-	-	-	-	-	1526	-	1526	0.753	Shared lane H/CD, upstream of exit
											Downstream of
Е	1149	-	1640	-	-	3	1640	0.820	1345	0.854	entry
G	689		1640				1640	0.820	1345	0.512	Upstream of entry

max x 0,854

In the second example, we consider the whole roundabout considering all the entries and exits in a consecutive way. That is, the queues from the downstream entry (G_k) will impede the upstream exit (H_k) . Thus, we have here a circular reference by the calculation which requires an iterative calculation. For the impedance from the entry to the exit (section HG) we assume that one vehicle $(n_{AE} = 1)$ can be stored between the exit and the entry (at the same arm). Obviously, the total intersection capacity will be smaller due to the circular interference of queuing. In this case we actually have a queuing sequence on the circular lane of the following sections:

$$H_1G_1 - E_1F_2 - H_2G_2 - E_2F_3 - H_3G_3 - E_3F_4 - H_4G_4 - E_4F_1 - (H_1G_1)$$

For a preloaded intersection volume of 1900 veh/h the degrees of saturations at different conflict points are illustrated in Table 11. One can recognize that the degrees of saturation are larger than those in example 1 as a consequence of the mutual queueing interactions on the circle.

com	sonseedrive way with encland queang interference with a total volume of 1900 ventil										
Cr	ross section			On circu	lar lane			At cor	respond	ing arm	
at arm k		a	C^*	down stream	section	C	r	a	C	r	
	ut ut tit h		C	n_{+1}	$f_{imp,+1}$	C	л	9	C	л	
1	Exit (F_1)	950	1386	$H_1G_1: n_{AE} = 1$	0.97	1343	0.71	570	1384	0.41	
	Entry (G ₁)	380	1640	E ₁ F ₂ : $n_{EA} = 3$	0.85	1390	0.27	570	853	0.67	
2	Exit (F ₂)	950	1416	$H_2G_2: n_{AE} = 1$	0.92	1297	0.73	380	1384	0.27	
	Entry (G ₂)	570	1640	E ₂ F ₃ : $n_{EA} = 3$	0.88	1438	0.40	380	973	0.39	
3	Exit (F ₃)	950	1386	H ₃ G ₃ : $n_{AE} = 1$	0.97	1343	0.71	570	1384	0.41	
	Entry (G ₃)	380	1640	E ₃ F ₄ : $n_{EA} = 3$	0.85	1390	0.27	570	860	0.66	
4	Exit (F ₄)	950	1416	H ₄ G ₄ : $n_{AE} = 1$	0.92	1297	0.73	380	1384	0.27	
	Entry (F ₄)	570	1640	E ₄ F ₁ : $n_{EA} = 3$	0.88	1438	0.40	380	973	0.39	
1	Exit (F ₁)	950	1386	$H_1G_1: n_{AE} = 1$	0.97	1343	0.71				
max x 0.73 0.6											
q	= traffic volu	ume of	the consi	dered cross sectio	n on the ci	rcular lane				[veh/h]	
$n_{\pm 1}$	= storage pla	aces on	the next	downstream section	on of circu	lar lane				[veh]	

Table 11 – Capacity of the conflict points on the circular lane considering all arms in the consecutive way with circular queuing interference with a total volume of 1900 veh/h

q	=	traffic volume of the considered cross section on the circular lane	[ven/n]
n_{+1}	=	storage places on the next downstream section of circular lane	[veh]
	=	n_{EA} (storage places between the entry and the downstream exit on the circular lane)	
	=	n_{AE} (storage places between the exit and the entry on the circular lane)	
C^{*}	=	capacity of the considered cross section without impedance of downstream queueing	[veh/h]
С	=	$C^* f_{imp+1}$	
	=	capacity of the considered cross section with impedance of downstream queueing	[veh/h]
f_{imp+}	1=	impedance factor of downstream queueing on the next section of circular lane (cf. eq. (47))	[-]

 f_{imp+1} = impedance factor of downstream queueing on the next section of circular lane (cf. eq. (47)) (e.g.: for the cross section F₁ section is C^* =1386, $f_{imp+1} = f_{imp}$ (H₁G₁)= $f(x_{G1})$ =1-0.27^(1.68×1+1)= 0.97, C=1386*×0.97=1343)

Again, by raising the total intersection volume, the total intersection capacity can be obtained. The results of this calculation are illustrated in Table 12. Obviously, none of the degrees of saturation on the circular lane can approach the value 1. Otherwise the whole roundabout will be totally gridlocked. According to the evaluation results, the example roundabout can only accommodate a critical total intersection volume of 1919 veh/h. This corresponds to an ADT of around 24000 veh/d (peak hourly volume = ADT * 0.08). Beyond this volume, the roundabout will be gridlocked and a breakdown will occur with no chance of recovery as long as the demand volumes remain constant. The gridlock effect becomes obvious by a steady increase of the x-values and a steady reduction of the capacities (down to zero) during the run of the iteration. At the critical total intersection volume, the maximum degree of saturation on

the circular lane in the example is only 0.77. The corresponding maximum degree of saturation at the approach entry and exits is 0.68.

Cross section				On circu	lar lane			At corresponding arm			
CI	at arm k		C^*	down stream	section	C	r	a	C	r	
	ut utilit it	Ч	C	$n_{\pm 1}$	$f_{imp,+1}$	C	л	9	C	л	
1	Exit (F_1)	960	1350	$H_1G_1: n_{AE} = 1$	0,96	1298	0,74	576	1384	0,43	
	Entry (G ₁)	384	1640	E ₁ F ₂ : $n_{EA} = 3$	0,79	1301	0,29	576	849	0,68	
2	Exit (F ₂)	960	1379	$H_2G_2: n_{AE} = 1$	0,90	1246	0,77	384	1384	0,28	
	Entry (G ₂)	576	1640	E ₂ F ₃ : $n_{EA} = 3$	0,84	1376	0,42	384	970	0,40	
3	Exit (F ₃)	960	1350	$H_3G_3: n_{AE} = 1$	0,96	1298	0,74	576	1384	0,43	
	Entry (G ₃)	384	1640	E ₃ F ₄ : $n_{EA} = 3$	0,79	1301	0,29	576	857	0,67	
4	Exit (F ₄)	960	1379	$H_4G_4: n_{AE} = 1$	0,90	1246	0,77	384	1384	0,28	
	Entry (F ₄)	576	1640	E ₄ F ₁ : $n_{EA} = 3$	0,84	1376	0,42	384	970	0,40	
1	Exit (F_1)	960	1350	$H_1G_1: n_{AE} = 1$	0,96	1298	0,74				
						max x	0,77			0,68	

Table 12 – Capacity of the conflict points on the circular lane considering all arms in the consecutive way with circular queuing interference with a total volume of 1919 veh/h

Certainly, the street flow split and the proportions of the turning movements have a strong influence of the total intersection capacity. To investigate this sensibility the street split was varied additionally in step 20%/80%, 30%/70%, 40%/60%, and 50%/50%. The proportion of the turning movement was varied correspondingly, that is 10%/80%/10%, 15%/70%/15%, 20%/60%/20%, and 25%/50%/25%. For a German single lane roundabout, the total intersection capacities with and without consideration of downstream exit impedance factor and circular queueing interference are illustrated in Table 13.

				0			
	Without dow	nstream exit	With down	stream exit	With downstream exit		
	impedance	factor and	impedance ia	ictor, without	impedance factor and		
	circular in	terference	circular in	terference	circular interference		
Street flow	Peak hour		Peak hour		Peak hour		
split	volume	ADT	volume	ADT	volume	ADT	
90/10	2457	30713	2078	25975	1875	23438	
80/20	2548	31850	2146	26825	1890	23625	
70/30	2647	33088	2218	27725	1907	23838	
60/40	2754	34425	2297	28713	1919	23988	
50/50	2870	35875	2385	29813	1924	24050	

 Table 13 – Total intersection capacities of a single lane roundabout with consideration of downstream exit impedance factor and circular queuing interference

It can be seen, that the misbalanced street split will cause a reduction of the total intersection capacity. Neglecting the downstream exit impedance factor and the circular queueing interference would lead to an overestimation of the intersection capacity. The total capacity obtained for the case with downstream exit impedance factor and circular interference delivers the most realistic total intersection capacities of single lane roundabouts under real world conditions (cf. the last two columns in Table 13.). However, the circular interference cannot be easily evaluated in practice due to the need for an iterative procedure which requires a useful computer program. Thus, at least the procedure with a consideration of the downstream exit impedance factor without circular interference is recommended for practical use. Here the working procedure in section 5 can be used step by step.

Analogously, the total intersection capacities of double lane roundabouts can be estimated as well. Details and examples for the calculation of the total intersection capacities of double lane roundabouts are not given here, they can be obtained from the authors.

7 Conclusion

This paper presents a model for capacity analysis of roundabouts with a completely new sophistication. It is based on the analysis of conflicts within the roundabout. The conflict points between the traffic streams of different types (cars and pedestrians) are considered by a homogenous model. The interaction between consecutive conflict points can be modeled according to the impedance probabilities. Together with this the distance between the consecutive conflict points is modeled properly.

Based on the proposed model, the capacity of the total roundabout can be analyzed more precisely. Especially, the interaction between different traffic streams and consecutive conflict points at roundabouts can be accurately taken into account according to the model. Using the proposed model, the whole roundabout can be treated as one entity and the total intersection capacity can be obtained according to given traffic volumes for the movements at the intersection. As a result the capacities of all the conflict points together with their degrees of saturation are obtained.

As one important result it becomes obvious that the interference of potential queuing processes between conflicts on the circular lanes can not be neglected, as it is the case for all the conventional roundabout capacity calculation methods. The current practice may lead to a significant overestimation of the total intersection capacity. Above a degree of saturation of x = 0.7 (obtained by conventional capacity estimation) a risk of queuing gridlock on the circle may occur.

The correct application of the derived equations may be rather complex. It is, however, not too problematic to implement them into a computer program.

To transfer the model to other countries a recalibration of the decisive parameters may be useful.

Further research may be directed on the influence of the b_{ij} (degrees of priority observation) between vehicles and in the vehicle/pedestrian conflicts. Additional empirical research should also analyze how these b_{ij} are depending on the saturation of the intersection.

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