A MODEL FOR AVERAGE SPEED ESTIMATION AND CRASH PREDICTION USING VEHICLE PATH DATA

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ABSTRACT

Research has shown roundabouts to significantly lower expected crash rates compared to traditional intersection designs. This improvement in safety is believed to be related to lower vehicle speeds and the more favourable angles between conflicting vehicle paths which lead to less severe crashes. Since modern roundabouts are designed to slow down traffic, speed control is essential to optimizing safety performance. It also stands to reason that the expected safety performance of a roundabout can be related to expected vehicle speeds.

Safety Performance Functions (SPFs) are statistically derived equations that predict the expected crash frequency of a site. SPFs have been developed both in the U.S. and internationally for roundabouts. However, the SPFs developed tended to include few variables useful for evaluating the safety impacts of alternate designs. This is perhaps not surprising given that such functions are in fact difficult to estimate, given that roundabouts tend to have very few crashes and design features with little variation. More recent research has modeled crashes at roundabouts as a function of vehicle speeds at various points in the roundabout. The premise being that if the safety performance of a roundabout can be related to vehicle speeds, and speed can be better estimated than crash experience can, then speed can be used indirectly as a surrogate in evaluating the safety implications of decisions in designing or re-designing a roundabout.

The database of roundabouts used for the current research included geometric, traffic, speed and crash data for 34 roundabout approaches. The speed data included the average vehicle speed for approaching, entry, upstream circulating and upstream exiting vehicles. The database was supplemented with the entry, circulating and exiting vehicle path radii for the design vehicle, the entry angle and the predicted vehicle speeds obtained from the TORUS Roundabout software.

In the present research, average vehicle speeds were modeled using the vehicle path radii predicted using TORUS Roundabout. It was found that the model predicting the average of entering, exiting and circulating speeds using vehicle path radii from TORUS Roundabout performs well. These predicted speeds could in turn be used with existing crash prediction models which use vehicle speeds to predict crashes.

INTRODUCTION

Modern roundabouts are increasingly being implemented as an alternative intersection design in North America. In other parts of the world roundabouts have been a popular intersection design for many years, in part due to their perceived road safety benefits. In fact, research has shown roundabouts to significantly lower expected crash rates compared to traditional intersection designs (Rodegerdts et al. 2007). This improvement in safety is believed to be related to lower vehicle speeds and the more favourable angles between conflicting vehicle paths which lead to less severe crashes. Since modern roundabouts are designed to slow down traffic, speed control is essential to optimizing safety performance. It also stands to reason that the expected safety performance of a roundabout can be related to expected vehicle speeds.

The field of traffic safety has evolved significantly in the past twenty years. The recent publication of the Highway Safety Manual (HSM), First Edition is evidence of this fact. Included in the HSM are chapters related to two-lane rural roads, multi-lane rural roads and urban and suburban arterials. These chapters include predictive methodologies for estimating the expected number of crashes at a location based on traffic exposure and roadway geometry and are intended to aid road designers in evaluating the safety effects of alternative design decisions. The basic approach is to apply a Safety Performance Function (SPF) for a set of base conditions (e.g. no turn lanes at an intersection) and then apply Crash Modification Factors (CMFs) where a sites geometry does not meet the base condition (e.g. a left-turn lanes exits). Safety Performance Functions are statistically derived equations that predict the expected crash frequency of a site. Crash Modification Factors may be either single values or equations which are multiplied by the SPF and are typically derived through before-after evaluations or cross-section studies of safety countermeasures. Separate SPFs and CMFs are available for roadway segments and intersections. At the present time, the HSM does not include a chapter for roundabouts although this is planned for future editions.

SPFs have been developed both in the U.S. and internationally for roundabouts. However, the SPFs developed tended to include few variables useful for evaluating the safety impacts of alternate designs. This is perhaps not surprising given that such functions are in fact difficult to estimate, given that roundabouts tend to have very few crashes and design features with little variation. More recent research has modeled crashes at roundabouts as a function of vehicle speeds at various points in the roundabout. The premise being that if the safety performance of a roundabout can be related to vehicle speeds, and speed can be better estimated than crash experience can, then speed can be used indirectly as a surrogate in evaluating the safety implications of decisions in designing or re-designing a roundabout.

REVIEW OF LITERATURE

Although the first edition of the HSM does not include a chapter on roundabouts there has been a significant amount of research on this topic. The state-of-the-art in roundabout safety research is to a large extent documented in NCHRP Report 572 (Rodegerdts et al. 2007). A literature review of existing non-U.S SPFs included in the report indicated that most involve relatively few independent variables as documented in Table 1.

Table 1 - International Roundabout Models from NCHRP Report 572

Country and Author	Sample Size	Constant Features	Variable Features	Model	Input Parameters
United Kingdom: Maycock and Hall	84	Four legs Single grade Circular island No unusual features	Island size speed	Total crashes at roundabout	AADT
				Total crashes at roundabout by crash type	AADT
				Total crashes at roundabout arm by crash type	AADT Pedestrian volume Entry width Entry angle Sight distance Approach curve Grade Radius Percentage of motorcycles in traffic
Australia: Arndt	100	None	Number of legs Number of lanes Urban/rural Island shape Speed	Total crashes at roundabout arm by crash type	AADT Number of lanes Speed variables Vehicle path radius Side friction
Sweden: Brude and Larsson	650	n/a	Number of legs Number of lanes Speed limit	Total crashes at roundabout	AADT
France: SETRA	n/a	n/a	n/a	Total crashes at roundabout	AADT

The NCHRP 572 report also presents SPFs developed using U.S. data at the roundabout level. The independent variables included in the roundabout level SPFs included entering average annual daily traffic (AADT), number of legs and number of circulating lanes. Data for individual roundabout approaches were also analyzed to develop SPFs that would be more useful for geometric design decisions. Separate

SPFs were developed for entering/circulating, exiting/circulating and approach crashes. However, the SPFs developed tended to include few variables useful for evaluating alternate designs.

The research documented in the NCHRP 572 report also attempted to establish a speed-based approach-level safety SPF using measured mean vehicle speeds with the following structure:

$$Crashes / year = \exp(\text{intercept}) \cdot AADT^{b} \cdot \exp(cX) \qquad (1)$$

Where,
AADT = average annual daily traffic
X = independent speed-related variable
Intercept, b, c = calibration parameters

The regression model was deemed inadequate on the basis of the weak effects of the speed variables so no speed-based SPF was recommended.

The research documented in NCHRP Report 572 showed that SPFs for roundabouts typically do not include many geometric variables that would allow a designer to assess the safety implications of decisions in designing a roundabout. An approach for addressing this void was explored by Chen, Persaud and Lyon (2011). The premise behind this research is that if the safety performance of a roundabout can be related to vehicle speeds, and speed can be better estimated from traffic and geometric variables than crash experience can, then speed can be used indirectly as a surrogate in evaluating the safety implications of decisions in designing or re-designing a roundabout. In this research four SPFs for predicting crashes were developed and compared based on the same sample of U.S. roundabouts used for the U.S. models in the NCHRP 572 report. The sample included data for the 33 individual approaches at 14 roundabouts which had measured average vehicle speeds. The four SPFs were a) approach level speed-based, b) approach level non speed-based, c) roundabout level speed-based, and d) roundabout level non speed-based. In developing the SPFs the average approach, entry, circulating and exiting speeds were considered. For the speed-based SPFs the best speed measure found for predicting crashes was the inside average speed (IAS) which averages the entry, upstream circulating and upstream exiting speeds for a given roundabout approach. For the roundabout level SPFs, the average of the IAS of each arm was used as the predictor speed variable. It was found that the SPFs including the IAS speed measures were superior to the SPFs lacking this variable in predicting crashes. The speed-based SPFs developed are shown below.

Speed-Based Approach Level Model

$$Y = \exp^{(-12.8046)}(AADT)^{0.8075}\exp^{(0.3388*IAS)} \qquad (2)$$

Where,
Y = total crash frequency for specific approach per year
AADT = entering AADT on approach
IAS = average of the entry, upstream circulating and upstream exiting mean speeds

in mph

Speed-Based Roundabout Level Model
$Y = \exp^{(-15.0165)}(AADT)^{1.0745}\exp^{(0.3260 * IASavg)}$ (3)

Where,
Y = total crash frequency for entire roundabout
AADT = entering AADT for entire roundabout
IAS_{avg} = average of the IAS for each approach

The research further developed models for predicting roundabout speeds as a function of design features, with a view to using the speeds estimated from these models, along with the speed-based SPFs, to assess roundabout safety performance. With this approach, speed is in effect used as a surrogate safety measure. The model developed is:

$IAS = 0.0253*ICD+0.1848*EW+9.51$ (4)

Where,
ICD = inscribed circle diameter in feet
EW = entry width in feet

Building on this work, Chen et al. (2012) developed geometric based models for predicting average vehicle speeds and then used these estimates to develop SPFs for roundabout approaches using a larger database of sites without in-field speed measurements. To develop the speed prediction models, the researchers used the same U.S. data as the previous study with additional data for roundabouts in Italy. The speed prediction model applied the IAS variable found in the earlier research to be the best predictor of roundabout crashes. The speed prediction model developed was:

$IAS = 13.015958 -3.088964*Cntry +0.034074*D_{av}+0.142936*W_{av}$ (5)

Cntry = 1 for U.S. site, 0 for Italian site
D_{av} = average of the inscribed circle and central island diameters in feet
W_{av} = average of the entry, circulating and exiting width in feet

The speed prediction model was then applied to a larger database of 139 U.S. roundabout arms to predict the IAS for each approach. The so predicted speeds were then used to develop a speed-based SPF as shown below.

$Crashes/year = \exp^{(-16.3755)}(AADT)^{0.5094}(IAS)^{4.3314}$ (6)

The authors concluded that the developed SPF suggests that the predicted speed approach seems to be promising for indirectly estimating roundabout safety performance. This approach is conceptually preferable than conventional observed speed based models for the advantage of expanding accessible sample size. The applicable sample size of U.S., for example, would be 33 if observed speed were applied to develop SPF. On the contrary, for predicted speed based modeling, sample size was 138.

The research by Chen et al. (2012) introduced the concept of a two-stage tool of roundabout safety analysis. The first stage is to predict average operating speeds, IAS, from geometric features and the second stage is to use the predicted IAS with an SPF for predicting expected crash rates. This approach offers promise for evaluating the safety of alternate roundabout designs given the difficulty of including geometry related variables directly into an SPF.

DESCRIPTION OF DATA

The database of roundabouts used for the present research was originally constructed for NCHRP Project 3-65 "Applying Roundabouts in the United States". The report for this project is NCHRP Report 572. The database includes geometric, traffic and crash data for 138 roundabout approaches, 34 of which have speed data. The speed data includes the average vehicle speed for approaching, entry, upstream circulating and upstream exiting vehicles. The geometric data includes:

- Number of circulating lanes
- Inscribed circle diameter
- Entry width
- Approach width
- Effective flare length
- Entry radius
- Entry angle
- Circulating width
- Exit width
- Departure width
- Exit radius
- Truck apron width
- Central island diameter
- Angle to next leg
- Presence of pedestrian crosswalk
- Presence of striping on circulating roadway

The traffic data includes:

- Entering AADT
- Upstream circulating AADT
- Exiting AADT

The crash data includes crash counts for the following crash types:

- Entry-rear end

- Approach rear-end
- Entry/circulating
- Circulating/exiting
- Circulating/exiting/rear-end
- Loss-of-control
- Vehicle-pedestrian
- Vehicle-bicycle

The database was supplemented with the entry, circulating and exiting vehicle path radii for the design vehicle and the entry angle determined by the TORUS software. In TORUS, the fastest paths are constructed from arcs that are tangent where they meet, and the speeds are functions of the arcs' radii. Figure 1 shows the various vehicle movements, R1 to R5. At a given location there are multiple fastest paths coming from the different approaches and overlapping therefore some assumption is necessary to combine these into an average predicted vehicle speed for entering, circulating and exiting vehicles. For entry and exiting speeds TORUS provides two speed estimates which are both considered. The 'alternate speed' estimate computes then entry and exiting speeds by evaluating the circulating speed, acceleration/deceleration rates and distance along the vehicle path. Both speed estimation methods are described in NCHRP Report 672 *Roundabouts: An Informational Guide, Second Edition.* Table 2 documents which fastest path measurements were considered for each speed estimate. The V1 to V5 speeds correspond to the R1 to R5 paths in Figure 1. If only one of the desired speed predictions was available then it was used. If more than one was available then an average was taken.

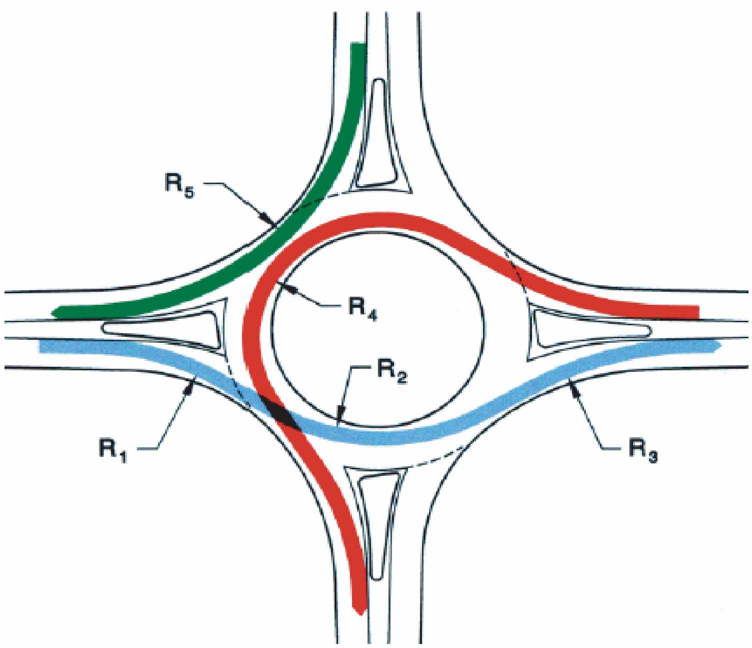

Figure 1 – Illustration of Vehicle Paths in TORUS

Table 2 – Predicted Speeds to Consider by Movement

Approach	Entry	Circulating	Exiting
North	North V1	East V2 South V4	South V3 East V5
South	South V1	West V2 North V4	North V3 West V5
East	East V1	South V2 West V4	West V3 South V5
West	West V1	North V2 East V4	East V3 North V5

SPEED PREDICTION

The modeling approach for speed prediction was to develop simple multi-variable linear regression models with the assumption that the error terms are normally distributed consistent with previous research. The model form was thus:

Predicted Speed = $a + b_1x_1 + \ldots b_nX_n$

Where,
a is the intercept term
b_1 to b_n are the other parameter estimates
x_1 to x_n are the independent variables

As per the previous research, the average of the entering, exiting and circulating speeds, denoted IAS, were modeled.

Prior to modeling, Pearson's Correlation Coefficients were determined between the potential independent and dependent variables. Pearson's Correlation Coefficients are a measure of the linear association between two variables. A value close to 1 or -1 indicates a high degree of correlation. If negative, this indicates that as one variable increases the second decreases. High degrees of correlation between independent and dependent variables indicate that an independent variable would be a strong predictor of the dependent variable. When independent variables are correlated caution should be taken when including both in the model because the estimated parameters may become unstable and statistically insignificant. Table 3 presents the correlation analysis between the potential independent variables and with the individual mean speed measurements. This information was used as a guide in developing the speed prediction models.

The independent variables which most consistently exhibit a high positive degree of correlation with the three speed measures are the inscribed circle diameter, central island diameter, circulating radius and circulating width. As would be expected, the degree of correlation between these same variables is also very high. This would indicate that these variables should be considered for inclusion in the speed prediction models but it is unlikely that they can all be included together.

Surprisingly, the correlation coefficients for entry and approach width have a negative correlation with the three speed measures indicating that as the lane width increases speeds go down. This counterintuitive result can be explained by the negative correlation between lane widths and the inscribed circle diameter and the vehicle path radii. As the inscribed circle diameter or path radii increase the lane width tends to be smaller. Therefore the larger lane widths tend to be for roundabout arms with a tighter path radii/smaller inscribed circle where speeds are lower.

Table 3 – Correlation Analysis Amongst Dependent and Independent Variables

	icd	cid	appwidth	entrywidth	exitwidth	circwidth	entryrad	circrad	exitrad	angle	entryspd	circspd	exitspd
icd	1.00	0.99	-0.40	-0.40	-0.31	0.84	0.27	0.99	0.32	0.20	0.57	0.86	0.82
cid	0.99	1.00	-0.47	-0.47	-0.40	0.83	0.34	0.97	0.35	0.14	0.64	0.87	0.81
appwidth	-0.40	-0.47	1.00	1.00	0.69	-0.29	-0.57	-0.40	-0.34	0.07	-0.46	-0.46	-0.38
entrywidth	-0.40	-0.47	1.00	1.00	0.69	-0.29	-0.57	-0.40	-0.34	0.07	-0.46	-0.46	-0.38
exitwidth	-0.31	-0.40	0.69	0.69	1.00	-0.03	-0.56	-0.36	-0.16	0.25	-0.41	-0.36	-0.12
circwidth	0.84	0.83	-0.29	-0.29	-0.03	1.00	0.24	0.74	0.53	0.23	0.37	0.68	0.78
entryrad	0.27	0.34	-0.57	-0.57	-0.56	0.24	1.00	0.26	0.34	-0.20	0.43	0.22	0.31
circrad	0.99	0.97	-0.40	-0.40	-0.36	0.74	0.26	1.00	0.25	0.18	0.59	0.86	0.78
exitrad	0.32	0.35	-0.34	-0.34	-0.16	0.53	0.34	0.25	1.00	-0.01	0.11	0.07	0.28
angle	0.20	0.14	0.07	0.07	0.25	0.23	-0.20	0.18	-0.01	1.00	-0.05	0.13	0.09
appspd	0.27	0.36	-0.38	-0.38	-0.36	0.07	0.22	0.31	-0.17	-0.05	0.67	0.53	0.38
entryspd	0.57	0.64	-0.46	-0.46	-0.41	0.37	0.43	0.59	0.11	-0.05	1.00	0.75	0.67
circspd	0.86	0.87	-0.46	-0.46	-0.36	0.68	0.22	0.86	0.07	0.13	0.75	1.00	0.83
exitspd	0.82	0.81	-0.38	-0.38	-0.12	0.78	0.31	0.78	0.28	0.09	0.67	0.83	1.00

icd = inscribed circle diameter in feet
cid = central island diameter in feet
appwidth = approach width in feet
entrywidth = entry width in feet
exitwidth = exit width in feet
circwidth = circulating width in feet
entryrad = entry path radius of the design vehicle in feet
circrad = circulating path radius of the design vehicle in feet
exitrad = exit path radius of the design vehicle in feet
angle = entering angle of the design vehicle in degrees
entryspd = average measured speed of entering vehicles in mph
circspd = average measured speed of circulating vehicles in mph
exitspd = average measured speed of exiting vehicles in mph

The first step in developing the speed prediction models was to include each independent variable on its own to assess its potential for inclusion in the final speed prediction model. Table 4 shows relevant information from these preliminary models. Each cell shows the R-squared value, the direction of effect on speed and a note if the parameter estimate was not statistically significant (n.s.) at the 90% confidence limit. The R-squared value is a goodness of fit measurement, with a value closer to 1 indicating a better fit of the model to the data. If the direction of effect is (+) that indicates than an increase in the value of the independent variable leads to an increase in predicted mean speeds, (-) indicates a decrease in predicted mean speed.

As would be expected based on the results of Table 3, the inscribed circle diameter, central island diameter, circulating radius and circulating width are the variables that provide the best predictive power. The next step was to attempt models with more than one independent variable.

Table 4 – Preliminary Speed Prediction Models Using TORUS Data

Variable	IAS
Inscribed circle diameter	0.69 (+)
Central island diameter	0.73 (+)
Entry radius	0.12 (+)
Exit radius	0.03 (+), n.s.
Circulating radius	0.68 (+)
Entry angle	0.005 (+), n.s.
Entry width	0.22 (-)
Exit width	0.09 (-)
Circulating width	0.47 (+)

The model form for the final speed prediction model is as follows:

Predicted Mean Speed in mph = intercept + b_1*circrad + b_2*entryrad + b_3*circwidth

Table 5 presents the estimated parameters, the standard errors of these estimates in parentheses and the R-squared value of the model.

Table 5 – Final Speed Prediction Models

Speed Measure	intercept	b_1	b_2	b_3	R-Squared
IAS	9.11674 (1.03868)	0.12502 (0.01698)	0.00817 (0.00656)	n/a	0.69

To assess the performance of the new speed prediction model for IAS compared to the existing models in equations (4) and (5), the predicted values from each model were plotted along with the observed value of IAS. Figure 2 plots these data with the observed speed on the x-axis and the predicted speed on the y-axis. It can be seen from Figure 2 that at very low values of IAS the predictions from the new model, denoted 'IAS pred', are a little further from the observed value of IAS. Conversely, at higher values of observed IAS the new model clearly predicts closer than the other available models.

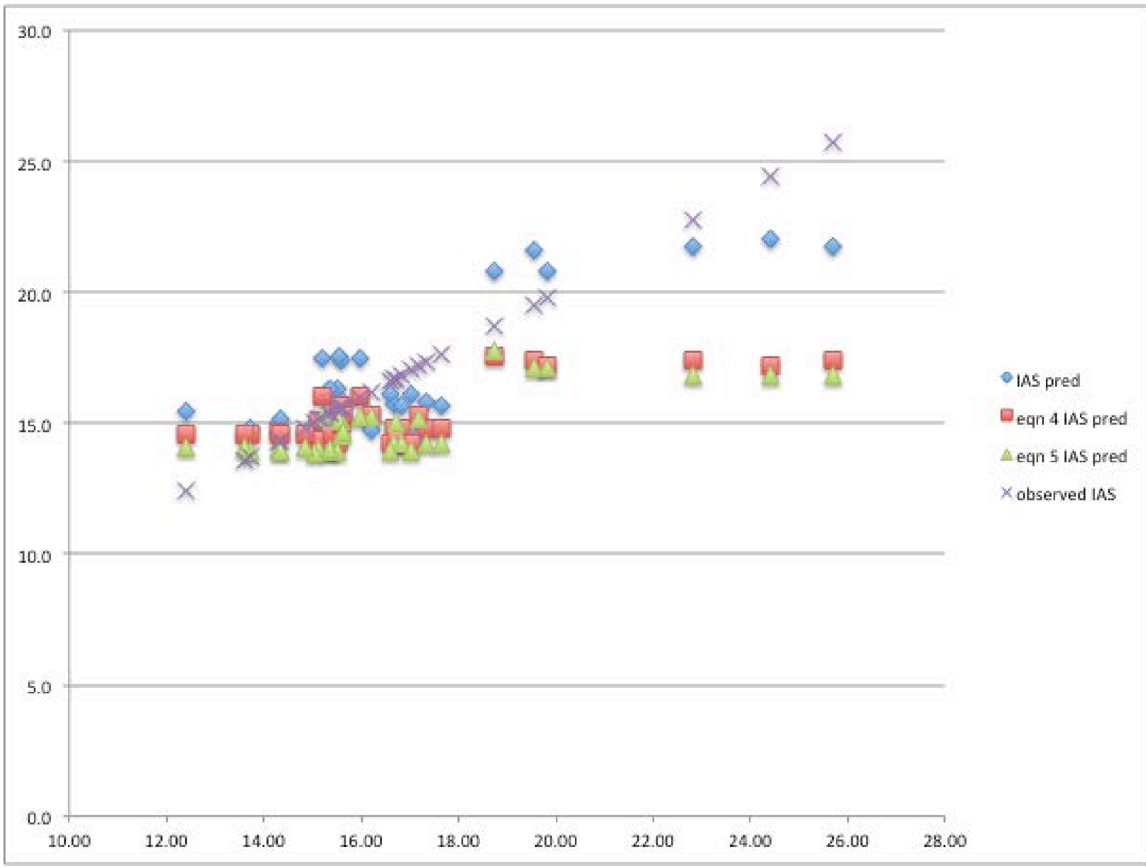

Figure 2 – Comparisons of IAS Prediction

Based on the comparison of model predictions for IAS it can be concluded that a speed prediction model using the vehicle path radii produced from TORUS performs equally well to the previous speed prediction models with geometric variables as independent variables.

CRASH PREDICTION

Consistent with state-of-the-art methods, generalized linear modeling, with the specification of a negative binomial (NB) error structure, was used to develop the crash prediction models. Alternative models were compared standard measures of goodness-of-fit such as the mean residuals (observed minus predicted) and the value of the overdispersion parameter which is estimated as part of the modeling process and is in itself a reliable goodness-of-fit measure, with a smaller overdispersion parameter indicating a model that better captures the overdispersion in the data.

The dataset for the new models contained one less roundabout arm which was not included because a speed estimate was not available for all movements. The new speed-based crash model has the same model form as equation (2) but now uses the vehicle speeds predicted by TORUS applied instead of the observed mean speeds. The model form is thus:

Speed-Based Approach Level Model

$Y = \exp^{(intercept)}(AADT)^{b1}\exp^{(b2*IAS)}$ (2)

Where,

Y = total crash frequency for specific approach per year

AADT = entering AADT on approach

IAS = average of the entry, upstream circulating and upstream exiting mean speeds in mph

Table 6 shows that the consistency in parameter estimates between the existing and new model are very close.

Table 6 – Comparison of New and Existing Crash Prediction Models

	intercept	**b_1**	**b_2**	**Overdispersion**
New Estimates	-12.9094 (3.5055)	0.8467 (0.3794)	0.3336 (0.0749)	0.9869
Existing Equation (2)	-12.8046 (2.838)	0.8075 (0.3045)	0.3388 (0.0587)	0.5568

What can be taken away from this is that the crash prediction models from equation (2) could be applied using the new speed prediction model for IAS which uses vehicle path radii as predictor variables. By the same logic, equation (6) which is based on a larger dataset and used predicted speeds in its calibration could also be applied.

CONCLUSIONS

Crash prediction models for roundabouts typically do not include many geometric variables that would allow a designer to assess the safety implications of decisions in designing a roundabout. An alternate approach whereby vehicle speeds are estimated using the roundabout layout and then these predicted speeds are used to predict expected crash rates has shown promise in previous studies.

The modeling of average vehicle speed using TORUS related vehicle path radii was successful. It was found that the new model predicting the average of entering, exiting and circulating speeds using vehicle path radii from TORUS performs equally well to the previous speed prediction models with geometric variables.

The suggested approach of predicting speeds based in part on vehicle path radii and then predicting crashes based on vehicle volumes and predicted speeds has advantages.

1. The roundabout level crash models from NCHRP Report 672 only use circulating lanes and number of approaches, so the approach using vehicle path radii to estimate speed is an improvement because the impact of roundabout geometry is better taken into consideration.
2. The approach level crash models from NCHRP Report 672 contain few geometric variables within each limiting their usefulness. The application of several models also complicates the analysis task.
3. Complex roundabouts such as those with spiral transitions, oval shapes etc. may be analyzed. Such non-standard geometries are not represented in geometry based crash models.

The speed and crash prediction models have several potential applications.

1. Estimating how safety changes when a design element changes such that vehicle speeds are affected.
2. Comparing the safety performance of an existing roundabout to the expected safety performance.
3. Evaluating the change in safety if a conventional intersection were to be replaced with a roundabout.

Applications 2 and 3 are safety management tasks which may be of interest to the same jurisdictions requiring the design work handled by application 1.

For application 2, comparing the safety performance of an existing roundabout to that expected, the observed crash history would be combined with the crash model estimates in the empirical Bayes (EB) procedure for estimating the long-term expected crash frequency of a site as is documented in the Highway Safety Manual.

For application 3, evaluating the change in safety of a roundabout conversion, crash prediction models for conventional intersections from the Highway Safety Manual could be applied. Or, the option for a jurisdiction to supply their own local models could also be available. Available crash data would be combined with the crash model estimates in the EB procedure.

Future research should aim to improve the accuracy of both the speed and crash prediction models. This can principally be accomplished by collecting more data for roundabouts. The sample size for the speed prediction models is particularly limited.

With an expanded database the development of crash type models should be re-visited which, if successful, would allow for the trade-offs between types of crashes to be accounted for in designing the roundabout layout.

The issue of using predicted speeds for each movement separately instead of the average may also be re-visited with an expanded dataset with an aim to improving the crash prediction models. Speeds for specific movements may be particularly important for specific crash types.

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